Leveraged ETFs, Holding Periods and Investment Shortfalls

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Leveraged and Inverse ETFs replicate the leveraged or the inverse of the daily returns of an index. Several papers have established that investors who hold these investments for periods longer than a day expose themselves to substantial risk as the holding period returns will deviate from the returns to a leveraged or inverse investment in the index. It is possible for an investor in a leveraged ETF to experience negative returns even when the underlying index has positive returns.

In this paper, we estimate distributions of holding periods for investors in leveraged and inverse ETFs. Using standard models, we show that a substantial percentage of investors may hold these short-term investments for periods longer than one or two days, even longer than a quarter.

We estimate the investment shortfall incurred by investing in leveraged and inverse ETFs compared to investing in a simple margin account to generate the same leveraged or short-selling investment strategy. We find that investors in leveraged and inverse ETFs can lose 3% of their investment in less than 3 weeks, an annualized cost of 50%. We also discuss the viability of leveraged and inverse leveraged ETFs that rebalance less often than daily and calculate their costs to investors.

I. Introduction

Exchange-Traded Funds (ETFs) are similar to index mutual funds but are listed and traded on exchanges similar to unit investment trusts and closed end mutual funds. Unlike mutual funds, that trade only once a day at net asset value, ETFs trade at varying prices throughout the day just like stocks.[2] State Street Global Advisors introduced the

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[2] For an in-depth discussion of the differences between index mutual funds and ETFs, see Guedj and Huang (2010).

first ETF in the United States – the SPDR, which tracks the S&P 500 index – in 1993.[3] Since 1993, investments in ETFs have grown rapidly, from $66 billion in 2000 to $2 trillion in 2010 and their underlying portfolios have expanded beyond domestic stocks into bonds, foreign stocks, and commodities.[4] Investments in ETFs now account for about 40% of the total amount invested in index mutual funds in the US. Many stock exchanges around the world now also list ETFs. iShares, State Street and Vanguard are the three largest issuers of ETFs.

Investors can leverage purchases in or sell short ETFs in margin accounts subject to the same margin rules that apply to purchases of most common stocks. Roughly speaking, the Federal Reserve Board's Regulation T prohibits the extension of credit to purchase common stock or the withdrawal of assets from a leveraged securities account that would reduce the investor's equity in the account below 50% of the value of the securities in the portfolio.[5] In addition, self-regulatory organizations ("SROs") require that member firms issue a margin call, i.e. demand additional unencumbered customer assets whenever the equity ratio in an account falls below a "maintenance requirement" of 25% because of changes in the market value of the securities held in the account.[6]

Leveraged and inverse ETFs combine traditional ETFs with internal borrowing or short selling to create simple leveraged or short investments. Until recently, investors in leveraged and short ETFs could make purchases that effectively leveraged or sold short an investment in securities without being constrained by margin rules.[7] For example, a leveraged ETF portfolio manager might borrow 200% of the equity in her portfolio and invest 300% of the equity value in securities. The equity in the ETF portfolio in that situation is only 33% of the securities value. An investor that concentrated her account in

[3] SPDRs - Standard & Poor's Depository Receipts or "Spiders" - are the largest ETF by market capitalization. The first ETF was introduced on the Toronto Stock Exchange in 1990.
[4] See ICI Fact book (2010)
[5] http://ecfr.gpoaccess.gov/cgi/t/text/text-idx?c=ecfr&sid=40cc031a4a064ca8d3f500270b0d0fd7&rgn=div8&view=text&node=12:3.0.1.1.1.0.1.12&idno=12
[6] In fact, many brokerage firms have maintenance requirements above 25%.
[7] FINRA NTM 09-53 (2009) announced higher margin requirements for leveraged and inverse ETFs that take into account the underlying leveraged or short market exposures. In addition to avoiding margin requirements, leveraged and inverse ETFs allowed investors to gain leveraged or short exposure in retirement accounts.

a 3-to-1 leveraged ETF would effectively be using leverage that would not be allowed in a retail margin account.

The portfolio manager of an inverse ETF effectively replicates short sales that could also be done in a retail margin account. The inverse ETF portfolio manager effectively borrows and sells short investments in the reference index, experiencing market returns opposite to the returns on the index and earning interest on the portfolio's cash balance. If the index's market return is negative and the net interest earned on the cash balance is positive, the inverse ETF will have a positive return.

Leveraged and inverse open-end mutual funds similar to leveraged and inverse ETFs had been in existence for many years prior to 2006. For example, ProFunds' UltraBull (ULPIX) and UltraBear (URPIX) open-end mutual funds, which leverage up and invert the daily returns to the S&P 500 respectively, were first offered in 1997. Like leveraged and inverse ETFs, these mutual funds rebalance their portfolios to re-establish their target exposure ratios at the end of each day.

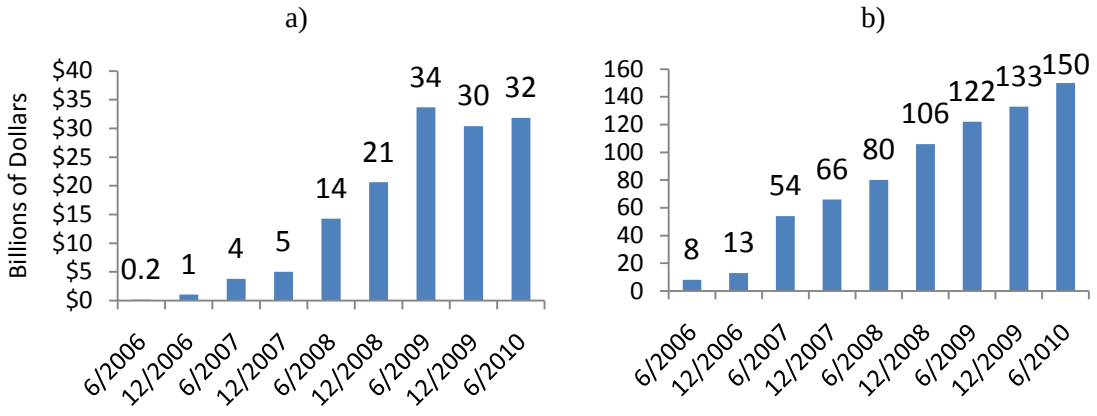
FINRA has issued a Notice to Members and additional guidance and the SEC has issued an Investor Alert about leveraged and inverse ETFs.[8] FINRA and the SEC have focused primarily on whether investors adequately understand that the returns to leveraged and inverse ETFs over holding periods longer than a few days are often significantly less than a multiple of the returns to the market index being referenced.

In this paper, we describe the problems associated with the daily rebalancing and the potential costs it may create for investors who hold these ETFs for longer than a few days. We use a methodology from the securities class action literature (see for example Barclay and Torchio (2001)) to infer the investors' holding periods from the observed trading volume. We apply this method to estimate the distribution of holding periods of investors in five different leveraged and inverse ETFs and use our results to calculate the shortfalls these investors have experienced compared to directly leveraging or selling short the underlying index with an ETF.

[8] FINRA Regulatory Notice 09-31 (2009), "Non-Traditional ETFs FAQ" at www.finra.org/Industry/Regulation/Guidance/P119781 and "Leveraged and Inverse ETFs: Specialized Products with Extra Risks for Buy-and-Hold Investors" at www.sec.gov/investor/pubs/leveragedetfs-alert.htm.

ProFunds issued the first leveraged and inverse ETFs in the United States in June 2006.[9] There were 13 leveraged and inverse ETFs at the end of 2006, 66 by the end of 2007, and 150 by June 30, 2010. The total market value of leveraged and inverse ETFs has grown from $1 billion in 2006 to more than $30 billion by 2010. See Figure 1.

Figure 1: Number of leveraged and inverse ETFs and assets under management from June 2006 to June 2010. Panel a) graphs the total market value (in billions of dollars) of all leveraged and inverse ETFs. Panel b) shows the number of leveraged and inverse ETFs.



The growth in investments in leveraged and inverse ETFs since 2006 has occurred in part because of investments made by or on behalf of unsophisticated investors. These investors may not understand that a 200% or 300% leveraged ETF doubles or triples the underlying index returns only over very short holding periods and that these leveraged ETFs are likely to return substantially less than double or triple the underlying index returns over holding periods longer than a few days or weeks. In fact, counter-intuitively, as a result of daily rebalancing of the leveraged and inverse ETF portfolios to re-establish the same leverage or short ratio at the end of each day, both 200% and 300% leveraged ETFs and inverse ETFs are quite likely to have negative returns across long holding periods whether the underlying market returns are positive or negative.

Table 1 lists the number of leveraged and inverse ETFs and market value by issuer as of June 30, 2010. The three primary issuers – ProFunds Group ("ProFunds"), Direxion Funds ("Direxion") and Rydex Investments ("Rydex") - are mutual fund companies that previously concentrated on active mutual fund traders and investment

[9] "ProFunds Readies ETFs That Leverage Indexes," Investor's Business Daily, 26 May 2006.

advisors. Together they account for 98% of the market capitalization of leveraged and inverse ETFs.

Table 1: Leveraged and inverse ETFs by issuer, as of June 30, 2010.

	Leveraged ETFs			Inverse ETFs			Leveraged ETFs	
	Number	Assets ($millions)		Number	Assets ($millions)		Number	Assets ($millions)
ProShares	42	$7,020		60	$18,379		42	$7,020
Direxion	17	$3,280		17	$2,290		17	$3,280
Rydex	7	$134		7	$148		7	$134
Other				3	$600			
Total	66	$10,434		87	$21,416		66	$10,434

Wang (2009), Cheng, Minder, and Madhavan (2009), Wong and Hargadon (2009), and Little (2010) show that daily rebalancing back to a specified leverage or short ratio requires leveraged and inverse ETF portfolio managers to buy at the end of days when the underlying market is up and sell at the end of days when the market is down.[10] When daily market returns are volatile but the realized returns over longer holding periods are close to zero, this rebalancing has the effect of repeatedly buying high and selling low. The more volatile the daily returns the greater the losses suffered by leveraged and inverse ETFs in compared to the leveraged or inverse returns to the market.

The paper proceeds as follows. Section II explains the mechanics of the daily portfolio rebalancing. We highlight the cost inherent in daily rebalancing using as examples, Direxion's Leveraged and Inverse Financial Services ETFs. Section III calculates the investment shortfalls incurred by unsophisticated investors. Section IV describes the investors' investment horizon and the possibility of developing investments that would be more suitable for their holding periods. We conclude in Section V.

II. Rebalancing, compounding and holding period returns.

Leveraged and inverse ETFs internally rebalance their long and short positions at the end of each day so that the leverage or short ratio is the same at the beginning of each day as it was at the initial public offering. Table 2 presents a simple, five-day example of

[10] See Zweig (2009) and Laise (2009) for discussions in the Wall Street Journal of the issue.

the impact of rebalancing and compounding on leveraged and inverse ETF returns. The daily returns accumulate over the five days to 0.01%.

Table 2: Example of the impact of rebalancing and compounding on ETFs.

	Index Returns		Traditional ETFs and Cash or Margin Debt			Leveraged and Inverse ETFs	
	a)	b)	c)	d)	e)	f)	g)
Day	Daily Return	Cumulative Return	Unlevered ETF	$200 cash, short $100 ETF	$200 margin, $300 ETF	1X I-ETF	3X L-ETF
0			$100.00	$100.00	$100.00	$100.00	$100.00
1	23%	23.00%	$123.00	$77.00	$169.00	$77.00	$169.00
2	-20%	-1.60%	$98.40	$101.60	$95.20	$92.40	$67.60
3	20%	18.08%	$118.08	$81.92	$154.24	$73.92	$108.16
4	-23%	-9.08%	$90.92	$109.08	$72.76	$90.92	$33.53
5	10%	0.01%	$100.01	$99.99	$100.03	$81.83	$43.59

Columns c), d) and e) reflect the returns that would be earned gross of fees and expenses in a traditional ETF, from selling short a traditional ETF and from leveraging up a traditional ETF 3-to 1. Column c) reports the value of $100 invested in an unlevered ETF at the beginning of the week is $100.01 at the end of the week. Column d) shows that the same $100 used to collateralize a short sale of $100 would end the week worth $99.99 ignoring any interest earned on the proceeds of the short sale. Column e) shows the result of leveraging up a traditional ETF 3-to-1 over the same 5-day period. We have constructed this example so that the 5-day holding period returns to the unleveraged ETF and to the leveraged and short investments in the ETF are all 0%.

Columns f) and g) report the results of selling short the underlying market by investing in an inverse ETF and the result of leveraging up the underlying market by investing in a 3X leveraged ETF. Column f) shows the result of investing $100 in an inverse ETF over the same week. Initially the inverse ETF portfolio holds a $100 short position in the index, $200 in collateral and $100 net equity. The resulting market exposure is -$100. On the first day, the market return is 23%, the inverse ETF's return is -23%, and the 1X inverse ETF investment is worth $77 since its underlying short position in the index is now $123 liability against its $200 in cash. The inverse ETF's market exposure is now -160% (i.e. -$123/$77 = -160%).

If the inverse ETF portfolio manager does not adjust the portfolio, the returns on the inverse ETF for the second day will equal -160% of the index return. To re-establish a -100% market exposure to start the second day, the portfolio manager uses $46 of the $200 in cash to reduce the -$123 end-of-day short market exposure down to -$77. The -20% index return on the second day reduces this short market exposure from -$77 to -$61.60 and the inverse ETF value increases to $92.40 at the end of the second day. At the end of the second day, because of the decrease in the value of the short position and the increase in the net asset value, the exposure ratio has fallen to -66.67%. To restore exposure to -100%, the inverse ETF manager must now increase back the leverage to -100% by increasing the short exposure by $30.80.

This simple example highlights the "constant leverage trap". Over a 5-day period, the ETF returns, as well as the leverage, and a short investment returns in the ETF was 0%. However, an investment in a 1X inverse ETF lost 18.2% and an investment in a 3x leveraged ETF lost 56.4%.

Cheng and Madhavan (2009) and Wang (2009) shows that the return on a leveraged ETF is:

$$(1 + R_T^{L-ETF}) = \left(1 + R_T^{index}\right)^x \cdot e^{\frac{(x-x^2)\sigma^2 T}{2}}$$

Where x is the leverage ratio, σ is the volatility of the index, and T is the time period the investment is held. For all leveraged ETFs in the market, the scalar term $e^{(x-x^2)\sigma^2 T/2}$ is positive, less than one and declines towards 0 the longer the holding period. The return of the leveraged ETF is a function of the return of the underlying index to the power of the leverage, multiplied by the scalar term. If the volatility is high enough and the holding period is long enough, the constant will be small and the return on the leveraged ETF will be smaller than that of its underlying index. It is possible for an investor in a leveraged ETF to earn negative returns even when the underlying index increases in value.

The daily rebalancing of leveraged and inverse ETFs creates a situation that for periods longer than a day or two the return of a leveraged or inverse ETF will deviate from the margin account benchmark. The magnitude of the deviation will depend on the

index characteristics for the holding period, mainly its volatility and its path. The higher the leverage and the longer the time period, the more likely it is that this deviation will be substantial. In general, as long as the underlying index has no clear trend, the higher the volatility, the higher the leverage, and the longer the time period, the more the investor will lose compared to investing in a leveraged or short position using a margin account.

The early ETF prospectuses did not fully explain the investment shortfall or warn that investors should exercise extra caution when investing in these funds. For example, one such prospectus stated, "*The Fund's current benchmark is 200% of the performance of the S&P 500 Index (the "Index" or "Underlying Index"). If the Fund meets its objectives, the value of the Fund's shares will tend to increase on a daily basis by 200% of the value of any increase in the Underlying Index.*"[11] Another prospectus stated that "*The correlations sought by the Bull Funds and the Bear Funds are generally a multiple of the returns of the target index or benchmark.*"[12]

Direxion issued (3X) and (-3X) ETFs in November 2008. Their September 29, 2008 prospectus stated, "*The Funds described in this Prospectus seek to provide daily investment results, before fees and expenses that correspond to the performance of a particular index or benchmark. The Funds with the word "Bull" in their name (collectively, the "Bull Funds") attempt to provide investment results that correlate positively to the return of an index or benchmark, meaning the Bull Funds attempt to move in the same direction as the target index or benchmark. The Funds with the word "Bear" in their name (collectively, the "Bear Funds") attempt to provide investment results that correlate negatively to the return of an index or benchmark, meaning that the Bear Funds attempt to move in the opposite or inverse direction of the target index or benchmark. The correlations sought by the Bull Funds and the Bear Funds are generally a multiple of the returns of the target index or benchmark.*" [13]

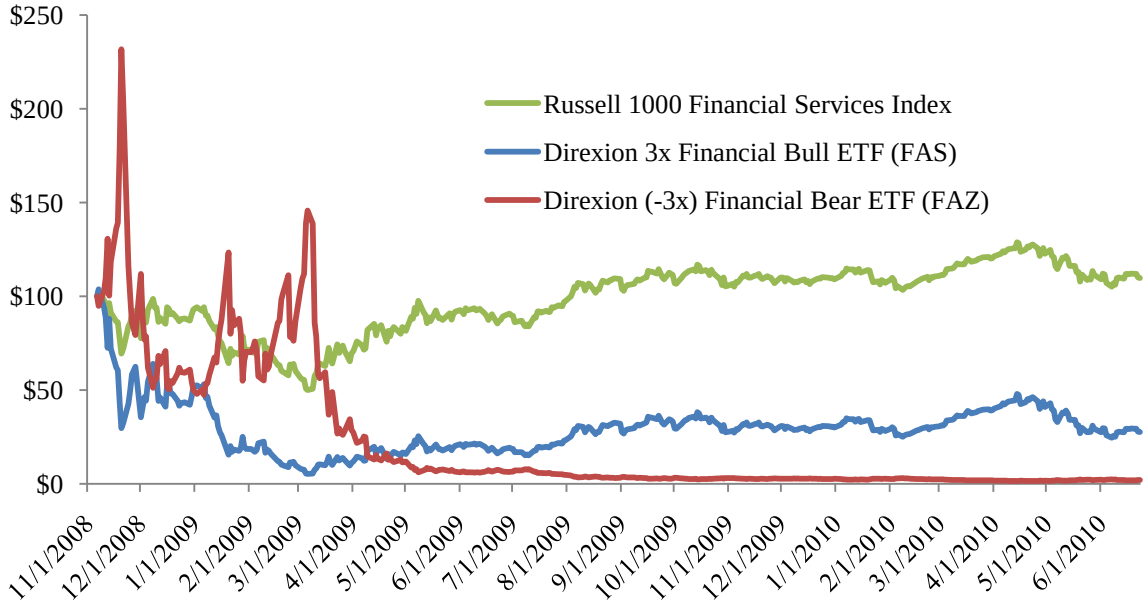
These statements illustrate how confusing descriptions of leveraged and inverse ETFs can be. The prospectuses did not always clearly explain that ETFs are not suitable for investors with investment horizons longer than one day. It was not until 2009 - after

[11] www.sec.gov/Archives/edgar/data/1208211/000093506906003020/g36000_etf485a.txt
[12] www.sec.gov/Archives/edgar/data/1424958/000089843208000978/direxion.htm
[13] www.sec.gov/Archives/edgar/data/1424958/000089843208000978/direxion.htm

many leveraged ETFs suffered significant losses while the reference ETFs had significant gains - that Rydex and ProShares ETFs improved their disclosures.[14] For example, Rydex's December 16, 2009 prospectus emphasized the *daily leveraged* investment goals and stated the leveraged ETFs were not suitable for "*investors who do not intend to actively monitor and manage their portfolios*." [15] ProShares' June 23, 2009 prospectus addressed investor suitability in a separate paragraph on two new products and then on all of the leveraged and inverse leveraged ETFs in their July 31, 2009 prospectus. [16]

Figure 2 plots the value of an investment of $100 in Direxion Financial Bull 3X ETF (FAS), Direxion Financial Bear 3X ETF (FAZ) and the Russell 1000 Financial Services Index (RGUSFL).[17] FAS and FAZ were first issued on November 6, 2008. FAS leverages up an investment in the financial services sector 3-to-1 each day, for one day. FAZ sells 300% of the fund's net assets short in the financial services sector each day, for one day.

Figure 2: Direxion's FAS, FAZ, and the Russell 1000 Financial Services Index from November 6, 2008 to June 23, 2010.



[14] Direxion emphasized on investor suitability in their prospectus filed on December 17, 2008.
[15] http://sec.gov/Archives/edgar/data/1208211/000089180409005431/sb47870-485b.txt
[16] http://sec.gov/Archives/edgar/data/1174610/000119312509135520/d485bpos.htm
http://sec.gov/Archives/edgar/data/1174610/000119312509160939/d485apos.htm
[17] As of May 28, 2010, RGUSFL 10 largest constituents were JPMorgan Chase, Bank of America, Wells Fargo, Citigroup, Goldman Sachs, US Bancorp, American Express, Morgan Stanley and Visa. www.russell.com/indexes/PDF/fact_sheets/US/1000Financialservices.pdf

Investors who thought that FAS or FAZ were effective ways to make any more than transitory bets on the direction of the financial services industry might be shocked by the returns illustrated in Figure 2. The Russell 1000 Financial Services Index gained 10% over the period reflected in Figure 2, yet FAS, the (3X) leveraged ETF, rather than returning 30% lost 72.4% and, the (-3X) inverse leveraged ETF, FAZ, rather than losing 30%, lost 97.9%. The counterintuitive pattern illustrated in Figure 2 is common for leveraged and inverse ETFs and results from the daily rebalancing of the funds' portfolios.

III. Potential Investment Shortfalls Incurred by Long-Term Investors

Unsophisticated investors who don't understand that leveraged ETFs are a poor way of leveraging or selling short an index for a period longer than a day or two may have experienced substantial investment shortfalls compared to having directly leveraged or shorted the underlying ETF in a margin account. The extent of the shortfall depends on the holding period of the investment and the returns and volatility of the underlying ETF. In order to precisely calculate the investment shortfalls caused by the mismatch between investors' investment horizon and the fund's daily horizon we need to observe actual holding periods. As these holding periods are not publicly available, we use trading models commonly used in establishing damages in securities class action litigation to estimate the holding periods. Barclay and Torchio (2001), Mayer (2000), McCann and Hsu (1999), and Beaver, Malernee and Keeley (1997) among others describe the methodology of using Trading Models and their advantages and shortcomings.

The simplest model, the Proportional Trader Model ("PTM"), assumes that each share outstanding is equally likely to trade. Thus, shares which trade each day are drawn from those which have recently traded and those which have not recently traded in proportion to the relative size of these two groups. For example, assume there are 1,000 shares outstanding and in one day we observe 200 shares traded. The PTM assumes that each investor sells proportionally 20% of their shares and are left with 80% of their previous day's holdings. If 100 shares are traded the next day, the PTM assumes that all investors – including investors who just bought the day before - sold 10% of their shares. The PTM repeats this process each day for the time period of interest and is thus able to

estimate the distribution of holding periods for each day's purchases. Murray and Belfi (2005) argue that the PTM method meets the legal criteria set by the Supreme Court for admission as a valid legal method for calculating damages.

The Multiple Trader Model ("MTM") assumes that there are at least two types of investors whithin each trader type with a different level of trading activity. Shares outstanding trade and daily trading volume are allocated among these types of traders and the PTM is applied to each type separately. The separate PTM results are then added together to arrive at total estimated damaged shares. Barclay and Torchio (2001) compare different variations of the proportional trading model to demonstrate that results from the proportional trading model can be consistent with the results of multi-trader models when certain assumptions and parameters are used. The MTM model appears to be appropriate for our research since a part of the ETF trades are done by market makers and arbitrageurrs and only a part is done by individual investors. See Appendix I for a detailed description of the procedures we follow.

We illustrate our methodology for estimating investment shortfalls with the five leveraged and inverse ETFs listed in Table 3. We use a cross section of ETFs from three different issuers, with a variety of positive and negative leverages, tracking a variety of indexes, including equity indexes, broad indexes, and bond indexes.

Table 3: List of five leveraged ETFs for which we calculate investment shortfalls.

Ticker	Name	Issuer	Leverage	Index
DPK	Developed Markets Bear 3X	Direxion	-3	MSCI EAFE
TYO	10-Year Treasury Bear 3X	Direxion	-3	NYSE 10 Year Treasury
RHO	Inverse 2X S&P Select Sector Health Care	Rydex	-2	AMEX Health Care Select
SBB	Short Small Cap 600 Fund	ProShares	-1	CBOE S&P Small cap 600
UVG	Ultra Russell 1000 Value Fund	ProShares	2	Russell 1000 Value

Table 4 reports the average turnover ratio for each ETF since inception and the estimated distribution of investors' holding periods. The average daily turnover ratio is an indicator of the average holding period. However, the MTM method allows us to estimate the distribution of holding periods.

As Table 4 illustrates, even leveraged and inverse ETFs that have a high daily turnover ratio will have some investors holding the ETF for longer than a few days. We

describe the holding period distribution by calculating the percentage of investors who hold the ETF for more than a week, a month, and a quarter. All five ETFs in our sample have a substantial percentage of holding periods longer than a month, ranging from 6% to almost 24% of the investors. More than 8% of the investors in SBB and UVG appear to hold the ETF longer than a quarter.

Table 4: Calculated holding periods for five leveraged ETFs.

ETF	Average Daily Turnover Ratio	Leverage Ratio	Average Holding Period (days)	Purchases Held for More Than 1 Week	Purchases Held for More Than 1 Month	Purchases Held for More Than 1 Quarter
DPK	18.1%	-3	5.3	16.42%	6.30%	1.22%
TYO	5.5%	-3	12.8	48.02%	16.39%	3.89%
SBB	4.6%	-1	21.4	55.49%	21.62%	8.50%
RHO	2.9%	-2	18.4	61.28%	27.62%	6.58%
UVG	3.7%	2	22.7	54.31%	23.91%	8.90%

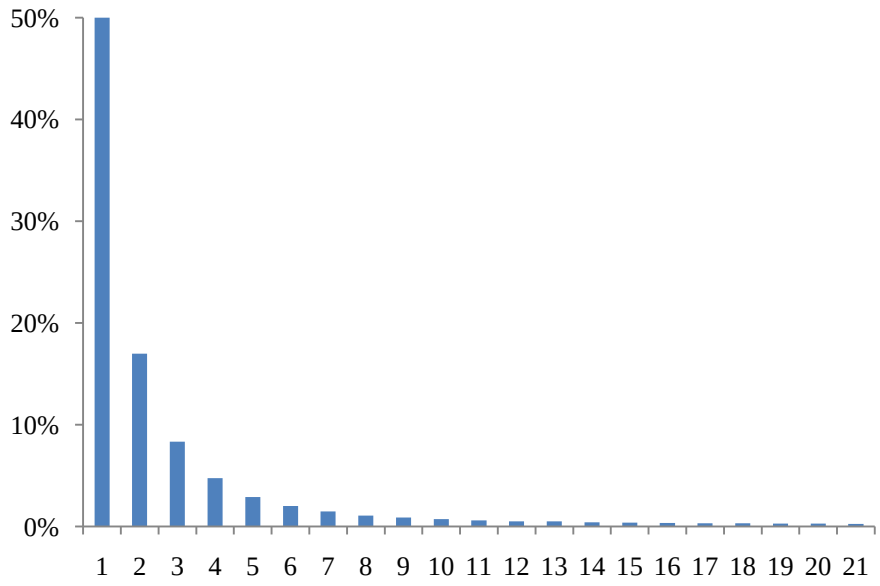
The daily turnover ratio is inversely related to the average holding period. SBB, RHO and UVG have lower turnover and higher average holding periods than DPK and TYO. However, the results of the distribution of the holding periods highlights the importance and contribution of using MTM method to infer the investors' holding periods. RHO has the lowest average daily turnover (2.9%) and its estimated average holding period (18.4 trading days) is not as long as UVG's estimated average holding period (22.7 days) even though UVG's daily turnover ratio is higher (3.7%). Nonetheless, more than 60% of purchased RHO shares are held longer than one week, while only 54% UVG shares purchased are held longer than a week. The MTM method gives us a tool to infer the distribution of holding periods, with is important when trying to analyze the behavior of investors in an investment designed to be held for only a short time.

The percentage of investors that we estimate hold these short term investments longer than a month is quite striking. More than 27% of RHO shares and more than 23% of UVG shares purchased appear to be held for longer than a month and more than 8% of SBB and UVG shares purchased appear to be held longer than a quarter - a very long time for an investment that should be held for only a day or two.

DPK has the highest daily turnover ratio and the lowest average holding period, as shown in Table 4. More than 16% of DPK shares purchased are held longer than one

week, more than 6% are held longer than a month, and more than 1% are held longer than a quarter. Figure 3 contains a histogram of the distribution of estimated investor holding periods of DPK. DPF's prospectus states "*the Funds are designed as short-term trading vehicles for investors who intend to actively monitor and manage their portfolios*."[18] Despite this statement, it appears that more than 6% of investors held this ETF longer than a month. This indicates that many investors do not understand the inherent cost associated with holding a short-term investment for a long-term.

Figure 3: Histogram of the holding period distribution for DPK. The horizontal axis is shown in trading days.



We use the MTM methodology to estimate the investment shortfall or the difference between holding the leveraged ETF and holding a leveraged investment in the index ETF in a margin account. In Figure 4, we present the difference between the two strategies' holding period returns as a function of the number of days the positions are held. For all holding periods, DPK on average had lower returns than the leveraged investment in the index ETF benchmark. Moreover, the longer the holding period, the greater the investment shortfall from the benchmark. On average, an investor that held DFK for 15 trading days lost 3% of her investment compared to the benchmark. In other words, had the investor created the leverage themselves in a margin account they would

[18] www.sec.gov/Archives/edgar/data/1424958/000089843210000945/a485bpos.htm, page 30.

have earned 3% more over a 3 week time period, the equivalent of more than 50% on an annualized basis.

Figure 4: Difference between the holding period return of the margin account and the leveraged ETF (Ticker: DPK), by length of holding period. The vertical axis shows how much higher the margin account's holding period return is relative to the leveraged ETF's. The horizontal axis shows the length, in trading days, that the position is held.

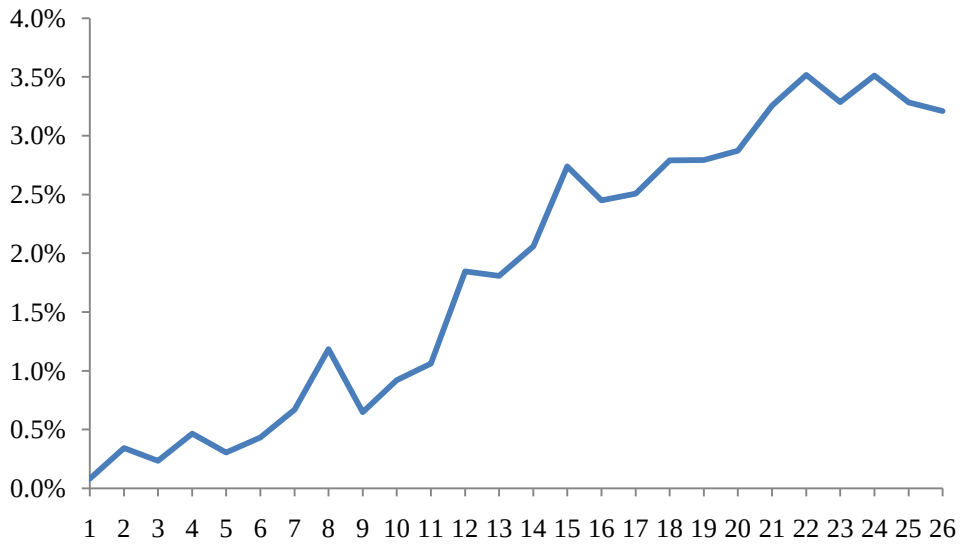


Figure 5 shows the total amount in dollar terms of the cummulative investment shortfall incurred by investors in DPK as a function of the number of days they held the ETF. We estimate that investors in DPK lost at least $1.8 million since the inception of the ETF in December 2008 compared to an investment in the benchmark portfolio. This amount is substantial, as DPK had a market capitalization of only $6 million at its inception and a subsequent average daily market capitalization of about $8.5 million.

Figures 4 and 5 illustrate two important facts about investors holding these ETFs for the long term. First, there can be a substantial investment shortfall for investors even when holding the ETF for only three or four days. On average, investors in DPK suffered a 0.5% investment shortfall over the first 4 days, more than 30% on an annualized basis. Second, investors holding DPK for up to four days account for $600,000 of the $1.8 million total shortfall.

Figure 5: Holding period cumulative total investment shortfall. The graph shows how much more the leveraged ETF (Ticker: DPK) would be worth if the ETF had been established using a margin account instead of being rebalanced daily. The horizontal axis is the number of trading days since the inception of the leveraged ETF.

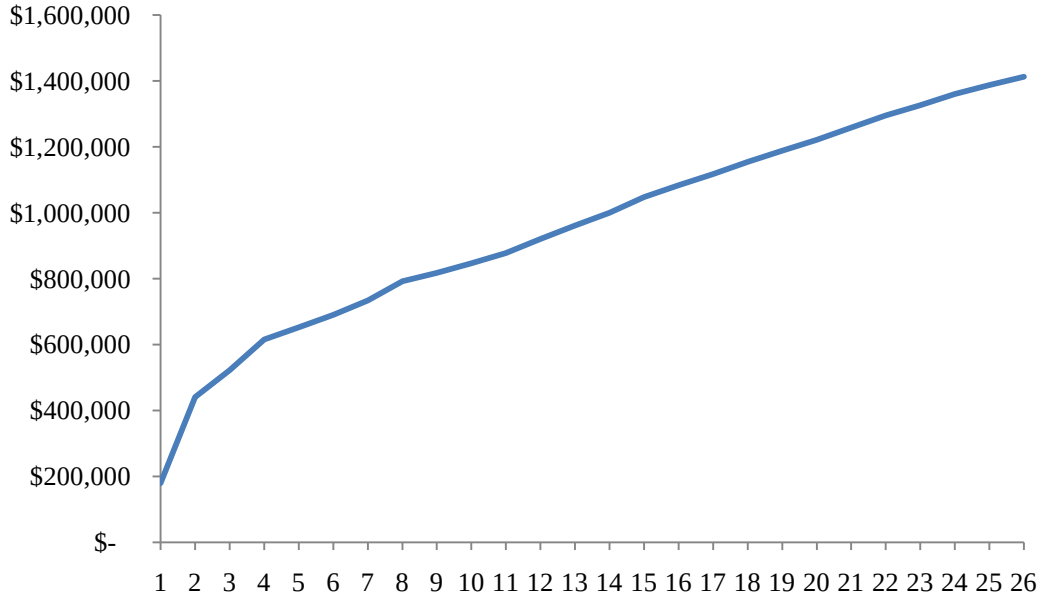


Table 5 reports the estimated investment shortfalls since inception and market capitalization of the five ETFs listed in Table 3. There is a large distribution of shortfalls between the different funds. On average, investors experienced an investment shortfall in each ETF. Due to the path-dependent nature of leveraged and inverse ETFs, an investment shortfall compared to directly leveraging or short selling an ETF in a margin account is not certain for every investor. However, our estimations indicate significant aggregate investment shortfalls in all of our case studies. The ubiquitous nature of the shortfalls illustrates the importance of ensuring that investors understand leveraged and inverse ETFs and their unique risks.

Table 5: Cumulative total investment shortfall of five leveraged ETFs. We estimate the aggregate investment shortfall from the ETF's inception through June 1, 2009.

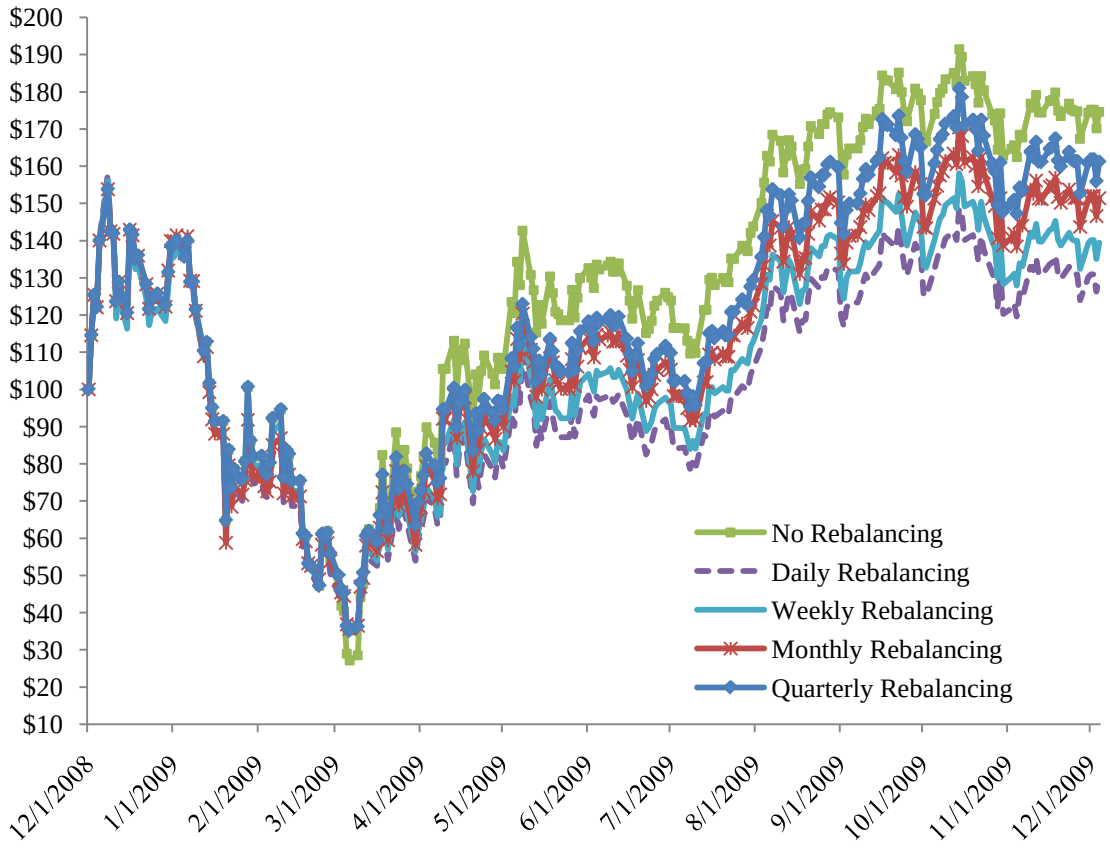
ETF	Leverage Ratio	Inception	Market Capitalization at Inception	Estimated Aggregate Shortfall
DPK	-3	12/17/2008	$ 6,046,000	$ 1,412,489
TYO	-3	4/15/2009	$ 6,100,000	$ 745,502
RHO	-2	6/12/2008	$ 7,802,000	$ 207,726
SBB	-1	1/25/2007	$ 15,630,750	$ 1,573,060
UVG	2	2/22/2007	$ 10,407,000	$ 464,699

IV. Investors' Investment Horizon and Funds' Rebalancing Frequency

New offerings by Direxion claim to match the investment horizon of investors better by rebalancing their portfolios only once a month. Little (2010) explains the concepts behind these investments.

Figure 6 plots the value of $100 leveraged 2-to-1 in an ETF that tracks the Dow Jones U.S. Financials Index in a margin account from December 1, 2008 to December 1, 2009. This investment is what we have been referring to above in our investment shortfall examples and calculations as the "benchmark". Figure 5 also plots the value of hypothetical 2-to-1 leveraged ETFs that rebalance daily, weekly, monthly and quarterly. As we can see in Figure 6, the less frequently leveraged ETFs rebalance their portfolios, the more closely their returns track the benchmark returns.

Figure 6: Comparison of holding returns using different compounding periods. The graph depicts the value of the DPK ETF (over time if it used daily, weekly, monthly, or quarterly rebalancing, or no rebalancing at all (equivalent to the margin account). DPK has 2-to-1 leverage.



Leveraged ETFs with a variety of rebalancing schedules may add value to investors, as they may be more suited to their needs but if a leveraged ETF rebalances monthly, investors buying in the middle of the month will invest at a time when the ETF's leverage might be dramatically different than its initial leverage. To be sure of its exposure, an investor would have to check what the ETF's leverage is on the day the investor intends to purchase unless it coincides with the date a rebalancing is performed.

Using our MTM methodology in order to calculate holding periods using trading volume data, we calculate the estimated shortfall from an investment in a theoretical ETF that rebalances monthly. The results of our calculations are presented in Table 6.

Table 6: Investment shortfalls in leveraged ETFs that rebalance monthly. We estimate the aggregate investment shortfall from the ETF's inception through June 1, 2009.

Name	Leverage Ratio	Inception Date	Investment Shortfall of ETF Rebalanced Daily	Investment Shortfall of ETF if Rebalanced Monthly
DPK	-3	12/17/2008	$ 1,412,489	$ 78,526
TYO	-3	4/15/2009	$ 535,768	$ (311,017)
RHO	-2	6/12/2008	$ 207,726	$ (12,389)
SBB	-1	1/25/2007	$ 1,573,060	$ (863,744)
UVG	2	2/22/2007	$ 464,699	$ 968,306

The results are surprising. Similar to the illustration in Figure 6, the investment shortfall is smaller with monthly rebalancing than with daily rebalancing for most but not all ETFs. However, the relationship between the two shortfalls does not always hold. DPK's shortfall nearly disappears as we change from daily rebalancing to monthly rebalancing, while UVG's shortfall doubles. The shortfalls for TYO, RHO, and SBB not only shrink, they turn negative as we change from daily rebalancing to monthly rebalancing. These results highlight that compared to investing using a margin account, even reducing the rebalancing frequency does not resolve the potential costs to investors looking to invest in leveraged or inverse positions in the long-run.

V. Conclusions

Cheng and Madhavan (2009) and Little (2010) argue that leveraged and inverse ETFs do not deliver the returns investors may expect when they invest in them for

periods longer than a day or two. FINRA has required the issuers of leveraged and inverse ETFs to caution their customers that these ETFs should be short-term investments and need to be monitored carefully.

In this paper, we follow this argument and investigate it further by estimating the distribution of the investors' holding periods in those ETFs from publicly available data. We find that many investors hold their leveraged ETFs for very long periods, at times longer than three months. Further, we calculate the shortfall of such a behavior compared to creating the leverage in a margin account. We find that some ETF investors lose up to 3% of their original investment in just a few weeks, the equivalent of a 50% annualized return. This indicates that investors do not fully understand the risks associated with inappropriately using leveraged and inverse ETFs as long-term investments.

Further, we investigate the value added to the marketplace by ETFs that rebalance monthly instead of daily. We find that the average investment shortfall is smaller but remains significant. Moreover, while we find less frequently rebalanced leveraged and inverse ETFs tend to have returns that are more similar to investing in a margin account, they may add risk as their leverage can vary significantly from day to day.

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Appendix I – The Use of Trading Models to Estimate Holding Periods

The standard trading models are explained by Barclay and Torchio (2001), Mayer (2000), McCann and Hsu (1999) and Beaver, Malernee and Keeley (1997). In this appendix, we provide a brief sketch of our application of the methodology.

The Proportional Trader Model ("PTM") assumes that each share outstanding is equally likely to trade. Thus, shares which trade each day are drawn from those which have recently traded and those which have not in proportion to the relative size of these two groups. This simple assumption is extraordinarily powerful since it can be used to generate distributions of holding periods.

Consider the example illustrated in Table A-1. There are 1 million shares outstanding and 100,000 shares are traded each day. Since each share traded is equally likely to trade in the PTM model, 10% of any shares still held from the 100,000 shares purchased on any given day are sold off each day thereafter.[19] That is, 10,000 of the 100,000 shares purchased on date *t* are subsequently sold on *t* + 1; 9,000 (10% of the 90,000 shares held longer than 1 day) are sold on *t* + 2; 8,100 are sold on *t* + 3 and so on.

Table A-1: Simple Trading Model Example

	PTM		MTM			
Shares Outstanding		1,000,000	200,000		800,000	
Daily volume		100,000	80,000		20,000	
	Still Held	Sold Off	Still Held	Sold Off	Still Held	Sold Off
t	100,000	0	80,000	0	20,000	0
t + 1	90,000	10,000	48,000	32,000	19,500	500
t + 2	81,000	9,000	28,800	19,200	19,013	488
t + 3	72,900	8,100	17,280	11,520	18,537	475
t + 4	65,610	7,290	10,368	6,912	18,074	463
…						
t + 10	34,868	3,874	484	322	15,527	398
…						
t + 20	12,158	1,351	3	2	12,054	309
t + 21	10,942	1,216	2	1	11,752	301
t + 22	9,848	1,094	1	1	11,459	294
t + 23	8,863	985	1	0	11,172	286
t + 24	7,977	886	0	0	10,893	279
t + 25	7,179	798	0	0	10,621	272

In the securities class action context this type of model is used to estimate how many shares purchased during an alleged fraud are held until the market learns the truth.

[19] These models can easily handle varying daily trading volumes and shares outstanding but require more assumptions than our simple example.

The same basic model provides a distribution of holding periods. In the simple example, 10% of the single day's purchases we have illustrated are held for 1 day, 6.6% are held for 5 days and 7.2% are held for more than 10 days. This same logic generates a distribution of holding periods for each day's purchases and these holding periods are aggregated up to create a distribution of holding periods for all the observed trading days.

Table A-1 also presents an MTM analysis for our example assuming two types of traders: a high activity type, which holds 200,000 shares and does 80% of the daily trading, and a low activity type, which holds 800,000 shares and does 20% of the daily trading. Consider, first, the active traders. They hold 200,000 shares and trade 80,000 shares each day. 40% of the active traders' shares are sold each day so 32,000 of the 80,000 share bought on date t are sold off on $t + 1$; 19,200 (40% of the 48,000 shares held longer than 1 day) are sold on $t + 2$; 11,520 are sold on $t + 3$ and so on.

The inactive traders' very low trading frequency means that some of the 20,000 shares purchased by this group on date t will be held for a long time. In fact, we can see in our example, although the MTM estimates a lot more shares than the PTM are held for only a few days, the MTM also estimates a lot more shares than the PTM are held for a long time.

Estimated holding periods allow us to estimate the investment shortfall for each day's purchases of an ETF using the ETF's daily closing prices. These investment shortfalls are then added up across all days to arrive at our estimate of the investment shortfall for the ETF.